

Name: Answer Key TP: \_\_\_\_\_

**Preparing for the final:**

You will be allowed to use ONE 8 ½ by 11 study guide on the final! You may only use ONE side (not both). You can hand write or type your study guide. Study guide MUST be approved by your geometry teacher BEFORE you take the final. If you fail to follow these directions you will NOT be allowed to use your study guide on the final! When in doubt, ask!

**What you can include:**

- Formulas/Rules
- Definitions
- Ideas/concepts
- Diagrams

**Not allowed:**

- Unit circle
- Worked out examples

Here are two examples:

<b>Example 1:</b>	<i>The volume of a cube is 64 inches cubed. What is the area of one face of the cube?</i> $V = s^3$ $64 = s^3$ $s = 4$ $A = 4 * 4 = 16$	<b>Allowed? Why or why not?</b>
<b>Example 2:</b>	$V(\text{cube}) = s^3$ <i>If given volume of a cube, remember to take cube root to find length of ONE side</i>	<b>Allowed? Why or why not?</b>

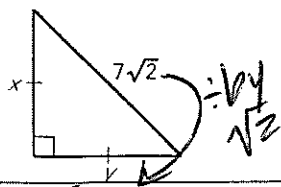
Here are the units we have studied 2<sup>nd</sup> semester. The review packet will follow in this order, as well as the final. Whatever helpful hints you think you might need, include it on your study guide!

- 1) Special Right Triangles
- 2) Composition of Functions & Undefined Functions
- 3) Trigonometry
- 4) Unit Circle
- 5) Complex Perimeter and Area
- 6) Properties of Quadrilaterals
- 7) Surface Area/Volume
- 8) Properties of Circles

Good Luck!

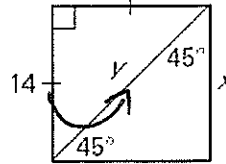
# Special Right Triangles

1) Find the value of each variable. Write answers in the simplest radical form.

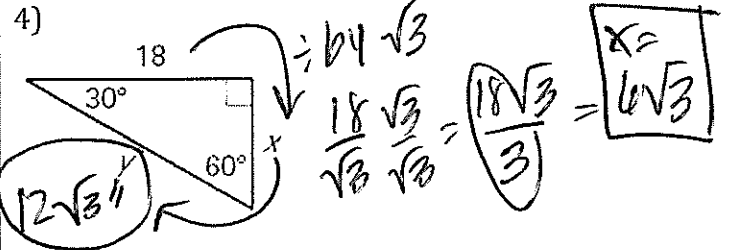
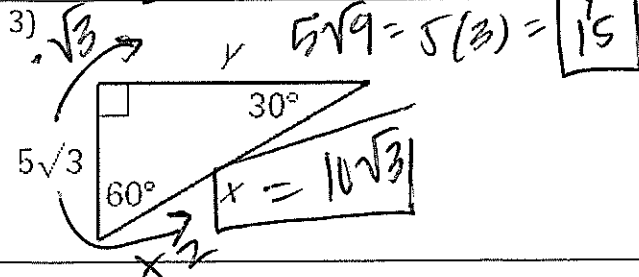


$$\frac{7\sqrt{2}}{\sqrt{2}} = 7$$

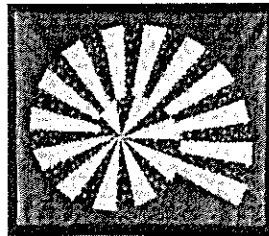
2) Find the value of each variable. Write answers in the simplest radical form.



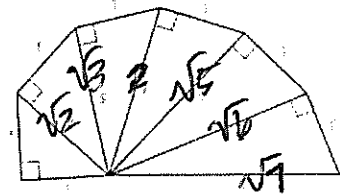
$$y = 14\sqrt{2}$$



The quilt design in the photo is based on the pattern in the diagram to the right. Use the diagram to answer the following questions.



Wheel of Theodorus



1. Find the values of r, s, t, u, v and w. Explain the procedure you used to find the values.

$$\begin{aligned} r &= \sqrt{2} \\ r^2 + (\sqrt{2})^2 &= 1 + 2 = \sqrt{3} \\ r^2 + \sqrt{3}^2 &= t^2 \\ 1 + 3 &= t^2 \\ \sqrt{4} &= t^2 \quad t = 2 \\ r^2 + 2^2 &= 4^2 \\ 1 + 4 &= \sqrt{5}^2 \\ \sqrt{5} &= u^2 \quad u = \sqrt{5} \end{aligned}$$

$$\begin{aligned} r &= \sqrt{2} \\ s &= \sqrt{3} \\ t &= 2 \\ u &= \sqrt{5} \\ v &= \sqrt{6} \\ w &= \end{aligned}$$

First I used 45-45-90 triangles & then Pythagorean theorem

\_\_\_ / 3

2. Which of the triangles, if any, is a 45° - 45° - 90° triangle? Which of the triangles, if any, is a 30° - 60° - 90° triangle? Explain how you know for each type of triangle. (Note: "r" can be used to reference the first triangle, "s" can be used to reference the second triangle, and so on.)

\_\_\_ / 4

The first triangle is a 45-45-90 because the ratio of the sides is 1:1:\sqrt{2}. The 3rd triangle is a 30-60-90 \Delta because the side ratio is 1:\sqrt{3}:2.

# Composition of Functions & Undefined Functions

- 3) If  $(f \circ g)(x) = \sqrt{x^2 + 10}$ , and  $g(x) = x^2 + 7$  what is a possible expression for  $f(x)$ ? Show why this works.

$$g(x) = x^2 + 7$$

$$f(x) = \sqrt{x+3}$$

$$f(g(x)) = \sqrt{x^2 + 7 + 3} = \sqrt{x^2 + 10}$$

- 4) If  $f(x) = \sqrt{4x}$  and  $g(x) = 7 - x$ , what is  $g(f(g(-9)))$ ?

$$g(x) = 7 - (-9) = 16$$

$$f(x) = \sqrt{4(16)} = 8$$

$$g(x) = 7 - (8) = -1$$

- 5) The regular price of a microwave in  $x$  dollars. Using a \$5 off coupon makes the price  $f(x) = x - 5$ . A 40% off sale makes the price  $g(x) = 0.6 \cdot x$ . Which composite function represents the better deal? Why?

microwave = \$100

A.  $(f \circ g)(x)$  - First take off \$5, then take off 40%

B.  $(g \circ f)(x)$  - First take off \$5, then take off 40%

C.  $(f \circ g)(x)$  - First take off 40%, then take off \$5

D.  $(g \circ f)(x)$  - First take off 40%, then take off \$5

take off \$5 first vs take off 40% first

$$100 - 5 = 95$$

$$95 \cdot 0.6 = 57$$

100  $\cdot$  0.6 = 60

- 6) If  $g(x) = 3x - 1$  and  $f(x) = x^2 + 1$ , compute  $f \circ g(x)$ .

$$(3x-1)^2 + 1$$

$$9x^2 - 6x + 1 + 1$$

$$9x^2 - 6x + 2$$

- 7) Compute  $(g \circ f)(x)$  using the functions  $f(x) = x^2 + 1$  and  $g(x) = 2x - 5$

$$2(x^2 + 1) - 5$$

$$2x^2 + 2 - 5$$

$$2x^2 - 3$$

- 8) If  $x$  is a real number, and the value of  $\frac{x}{x+5}$  is undefined, what is the value of  $x + 8$ ?

$$x + 5 = 0$$

$$x = -5$$

$$-5 + 8 = 3$$

- 9) What is the product of the values that make the following expression undefined?

$$\frac{4}{z^2 + 9z + 14}$$

$$(z+2)(z+7)$$

$$z = -2, -7$$

$$(-2)(-7) = 14$$

- 10) The expression  $\frac{5-x}{\sqrt{x}}$  is undefined when  $x$  is:

can't be 0!

- 11) The expression  $\frac{4+x}{\sqrt{4x+20}}$  is undefined when  $x$  is:

$$4x + 20 < 0$$

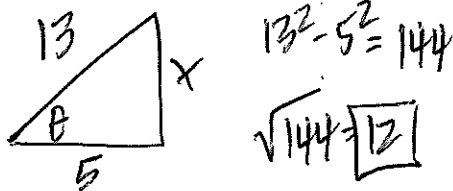
$$4x < -20$$

$$x < -5$$

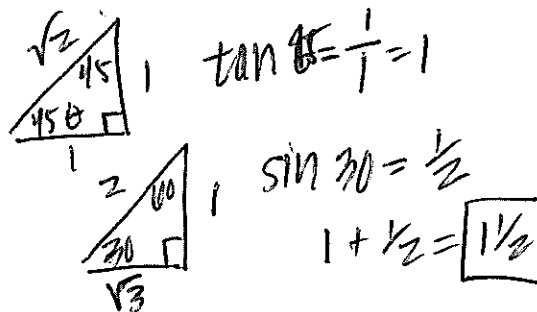
# Trigonometry

12) If in a right triangle  $\tan \theta = \frac{x}{5}$  and  $\sin \theta = \frac{x}{13}$ , then  $x = ?$

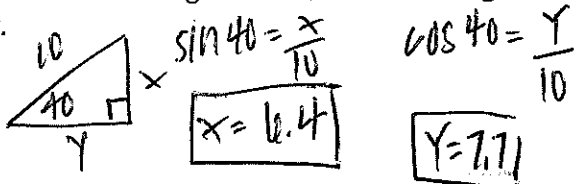
- A. 1
- B. 7
- C. 12
- D. 144
- E. Cannot be determined from given information



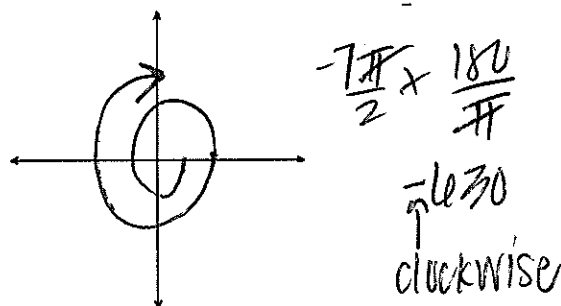
13) Find the **exact** value of  $\tan \frac{\pi}{4} + \sin 30^\circ$ . **Prove** your answer using special right triangles.



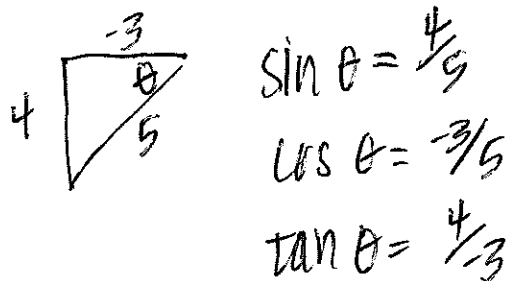
14) A right triangle has a hypotenuse of length 10 centimeters. If one angle is  $40^\circ$ , find the length of each leg.



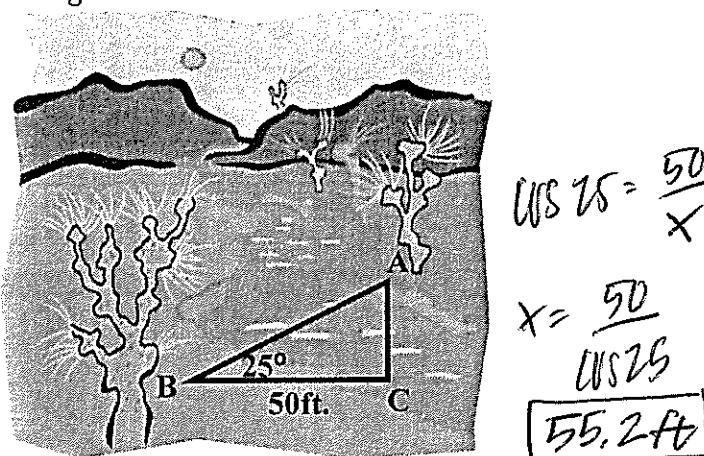
15) Draw an angle that measures  $-\frac{7\pi}{2}$  radians.



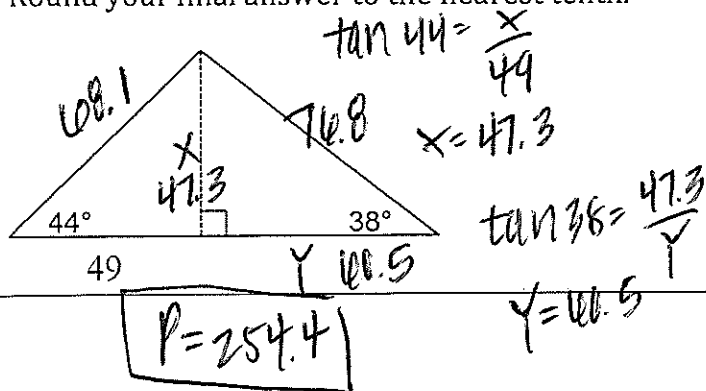
16) Find the exact value of each of the three remaining trigonometric functions of an angle  $\theta$  if  $(-3, 4)$ , is a point on the terminal side of  $\theta$ .



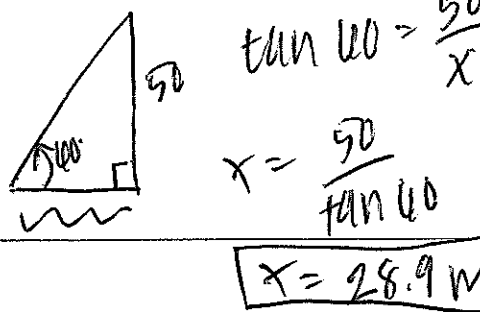
17) **Finding the Width of a River** Find the distance from A to C across the river illustrated in the figure.



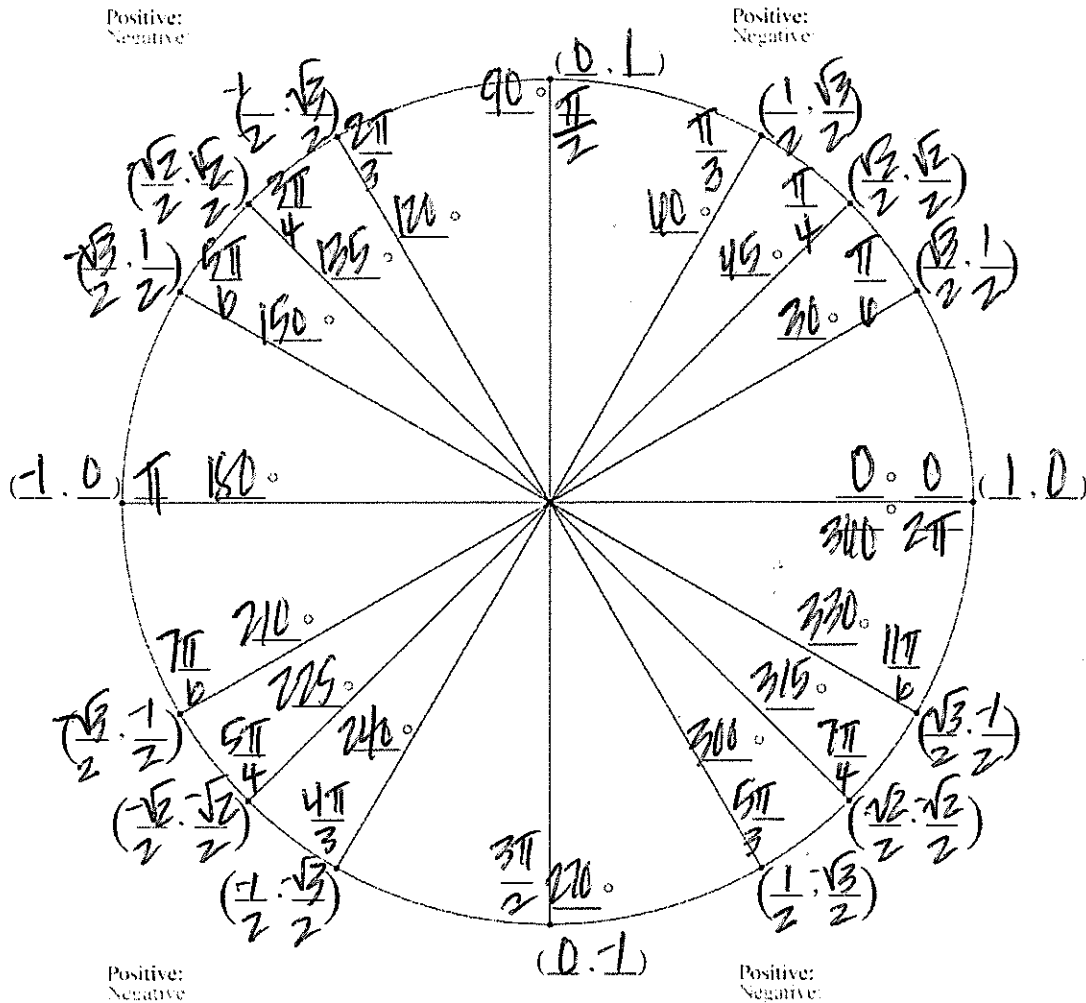
18) Find the perimeter of the larger triangle. Round intermediate values to the nearest tenth. Use the rounded values to calculate the next value. Round your final answer to the nearest tenth.



19) Suppose that you are headed toward a plateau 50 meters high. If the angle of elevation to the top of the plateau is  $60^\circ$ , how far are you from the base of the plateau? Draw a picture and round your answer to the nearest tenth.

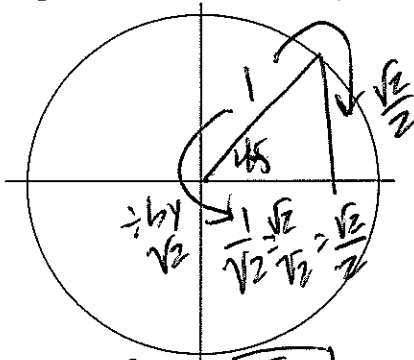


# Unit Circle



Use right triangles to derive the coordinate points in the first quadrant of the unit circle.

1)  $\theta = 45^\circ$  (draw a special right triangle for each unit circle)

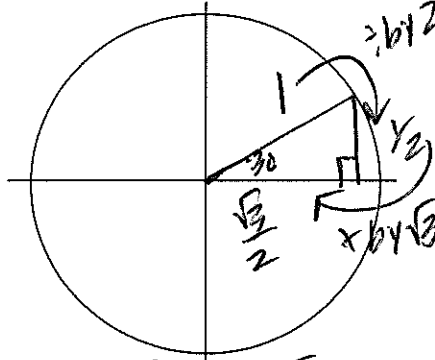


$$\cos 45^\circ = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$$

$$\sin 45^\circ = \frac{\frac{\sqrt{2}}{2}}{1} = \frac{\sqrt{2}}{2}$$

$$\text{Coordinate} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

2)  $\theta = 30^\circ$  (draw a special right triangle for each unit circle)

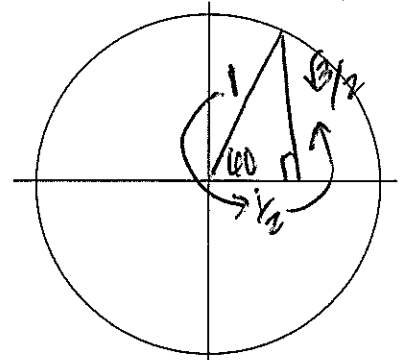


$$\cos 30^\circ = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

$$\sin 30^\circ = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

$$\text{Coordinate} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

3)  $\theta = 60^\circ$  (draw a special right triangle for each unit circle)

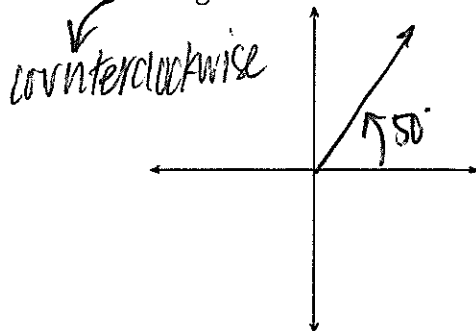


$$\cos 60^\circ = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

$$\sin 60^\circ = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

$$\text{Coordinate} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

4) Draw a  $50^\circ$  angle.



a. Find **two** positive angles that are coterminal with the given angle.  $50 + 360 = 410$

$$410 + 360 = 770$$

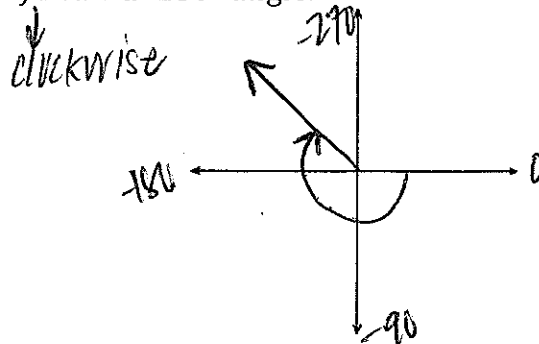
b. Find **two** negative angles that are coterminal with the given angle.  $50 - 360 = -310$

$$-310 - 360 = -670$$

c. Convert  $50^\circ$  to radians.

$$\frac{50^\circ}{1} \times \frac{\pi}{180} = \boxed{\frac{5\pi}{18}}$$

5) Draw a  $-230^\circ$  angle.



a. Find **two** positive angles that are coterminal with the given angle.  $-230 + 360 = 130$

$$130 + 360 = 490$$

b. Find **two** negative angles that are coterminal with the given angle.  $-230 - 360 = -590$

$$130 - 360 = -230$$

c. Convert  $-230^\circ$  to radians.

$$\frac{-230^\circ}{1} \times \frac{\pi}{180} = \boxed{-\frac{23\pi}{18}}$$

6) Convert  $\frac{3\pi}{2}$  radians to degrees.

$$\frac{3\pi}{2} \times \frac{180}{\pi} = \boxed{270^\circ}$$

7) Convert  $\frac{4\pi}{3}$  radians to degrees.

$$\frac{4\pi}{3} \times \frac{180}{\pi} = \boxed{240^\circ}$$

NAME	ABBREVIATION	RATIO	NAME	ABBREVIATION	RATIO
sine	$\sin$	$\frac{o}{h}$	cosecant	$\csc$	$\frac{h}{o}$
cosine	$\cos$	$\frac{a}{h}$	secant	$\sec$	$\frac{h}{a}$
tangent	$\tan$	$\frac{o}{a}$	cotangent	$\cot$	$\frac{a}{o}$

8) Evaluate the six trigonometric functions of the angle  $\theta$ .

$6^2 + 8^2 = 10^2$   $x^2 = 100$   $x = 10$

$$\sin \theta = \frac{6}{10} \quad \csc \theta = \frac{10}{6}$$

$$\cos \theta = \frac{8}{10} \quad \sec \theta = \frac{10}{8}$$

$$\tan \theta = \frac{6}{8} \quad \cot \theta = \frac{8}{6}$$

9) Let  $\theta$  be an acute angle of a right triangle. Find the value of the other five trigonometric functions of  $\theta$ .

$\sqrt{2}^2 + x^2 = 2$

$$\sin \theta = \frac{\sqrt{2}}{2} \quad \csc \theta = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2} \quad \sec \theta = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\tan \theta = \frac{\sqrt{2}}{\sqrt{2}} = 1 \quad \cot \theta = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

10) Evaluate the 6 trig functions of  $45^\circ$

$$\sin 45^\circ = \frac{\sqrt{2}}{2} \quad \csc 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2} \quad \sec 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = 1 \quad \cot 45^\circ = 1$$

11) Evaluate the 6 trig functions of  $30^\circ$

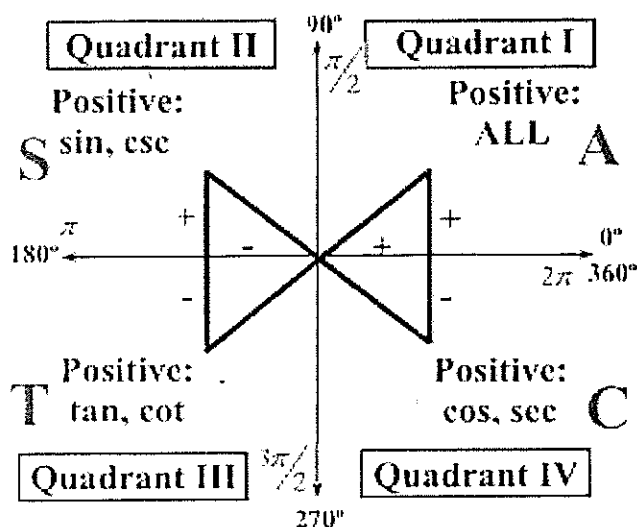
$$\sin 30^\circ = \frac{1}{2} \quad \csc 30^\circ = \frac{2}{1} = 2$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \sec 30^\circ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad \cot 30^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

If  $\theta$  is not a quadrantal angle, the sign of a trig function depends on the quadrant in which  $\theta$  lies:

Draw Picture:



"A Smart Trig Class" or "All Students Take Calculus"

A S T C

\*All positive in Q1

\*Sine (and reciprocal function cosecant) positive in Q2

\*Tangent (and reciprocal cotangent) positive in Q3

\*Cosine (and reciprocal secant) positive in Q4

12) Evaluate, if possible, the sine function and tangent function of the 4 quadrantal angles:

a.  $\theta = 0^\circ = 0$   $\sin \theta = 0$   
 $\tan \theta = \frac{0}{1} = 0$

b.  $\theta = 90^\circ = \frac{\pi}{2}$   $\sin \frac{\pi}{2} = 1$   
 $\tan \frac{\pi}{2} = \frac{1}{0} = \text{undefined}$

c.  $\theta = 180^\circ = \pi$   $\sin 180 = 0$   
 $\tan 180 = \frac{0}{-1} = 0$

d.  $\theta = 270^\circ = \frac{3\pi}{2}$   $\sin 270 = -1$   
 $\tan 270 = \frac{-1}{0} = \text{undefined}$

13) Name the quadrant in which angle  $\theta$  lies.

a.  $\sin \theta < 0$  and  $\cos \theta < 0$

$\begin{array}{c|c} S \times & A \\ \hline T \times & C \end{array}$  Q3

b.  $\tan \theta < 0$  and  $\cos \theta < 0$

$\begin{array}{c|c} S \checkmark & A \\ \hline T \times & C \end{array}$  Q2

c.  $\tan \theta < 0$  and  $\sin \theta < 0$

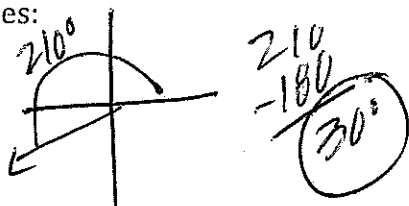
$\begin{array}{c|c} S \checkmark & A \\ \hline T \checkmark & C \end{array}$  Q4

d.  $\cot \theta > 0$  and  $\sec \theta < 0$

$\begin{array}{c|c} S \checkmark & A \\ \hline T \checkmark & C \end{array}$  Q3

14) Find the reference angle  $\theta'$ , for each of the following angles:

a.  $\theta = 210^\circ$

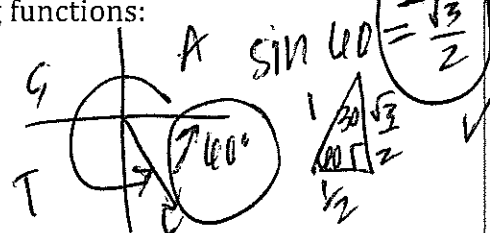


b.  $\theta = \frac{7\pi}{4}$

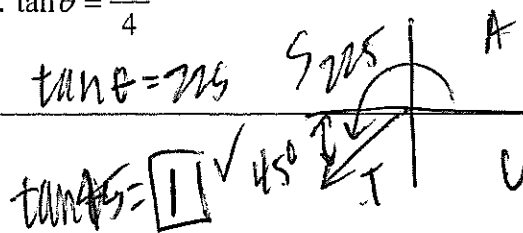
$\frac{7\pi}{4} \times \frac{180}{\pi} = 315^\circ$

15) Use reference angles to find the exact value of the following trig functions:

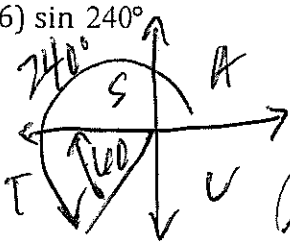
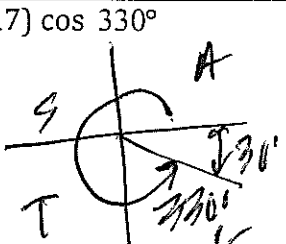
a.  $\sin \theta = 300^\circ$



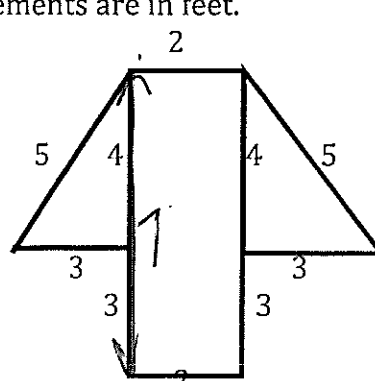
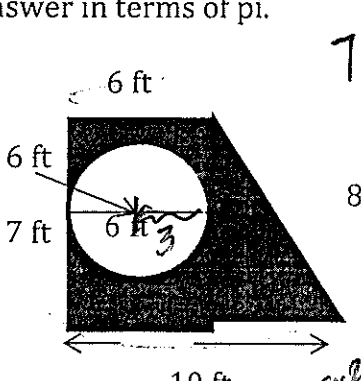
b.  $\tan \theta = \frac{5\pi}{4}$



Evaluate the function without using a calculator.

<p>16) <math>\sin 240^\circ</math></p>  <p><math>\sin 60 = \frac{\sqrt{3}}{2}</math></p>	<p>17) <math>\cos 330^\circ</math></p>  <p><math>\cos 30 = \frac{\sqrt{3}}{2}</math></p>
<p>18) <math>\sin \frac{4\pi}{3}</math></p> <p><math>\sin 240</math> same</p>	<p>19) <math>\cos \frac{7\pi}{6}</math></p> <p><math>\cos 210</math></p> <p><math>\cos 30 = \frac{\sqrt{3}}{2}</math></p>
<p>20) Solve the equation for <math>\theta</math> without a calculator. Give your answer in both radians and degrees.</p> <p><math>\sin \theta = (-\frac{\sqrt{3}}{2})</math></p> <p><math>\theta = \sin^{-1}(-\frac{\sqrt{3}}{2})</math></p> <p><math>120^\circ</math> or <math>240^\circ</math> restricted domain</p>	<p>21) Solve the equation for <math>\theta</math> without a calculator. Give your answer in both radians and degrees.</p> <p><math>\tan \theta = (-\sqrt{3})</math></p> <p><math>\tan^{-1}(-\sqrt{3}) = -60^\circ</math></p> <p>restricted domain <math>300^\circ</math></p>
<p>22) Solve the equation for <math>\theta</math> without a calculator. Give your answer in both radians and degrees.</p> <p><math>\cos \theta = (-\frac{1}{2})</math></p> <p><math>\cos^{-1}(-\frac{1}{2})</math></p> <p><math>120^\circ</math> or <math>240^\circ</math> restricted domain</p>	<p>23) Solve the equation for <math>\theta</math> without a calculator. Give your answer in both radians and degrees.</p> <p><math>\sin \theta = 0</math></p> <p><math>\sin^{-1}(0) = 0^\circ</math></p>

### Complex Perimeter & Area

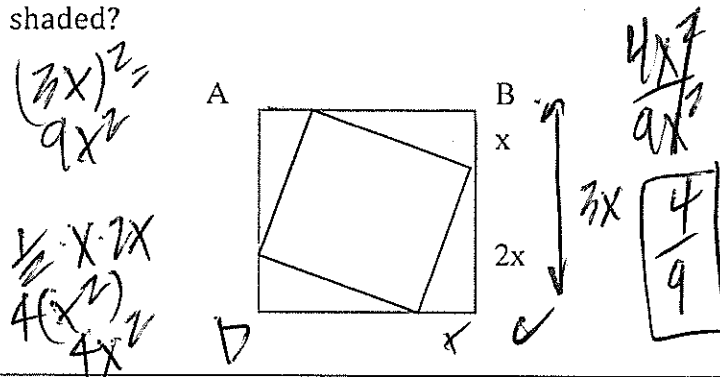
<p>24) What is the area of the entire hexagon? All measurements are in feet.</p>  <p><math>3(4) = 12</math>  <math>7(2) = 14</math>  <math>26 \text{ ft}^2</math></p>	<p>25) What is the area of the shaded region? Leave your answer in terms of pi.</p>  <p><math>7(6+10)</math>  <math>7(16)</math>  <math>56</math>  <math>-9\pi</math>  <math>22.7 \text{ ft}^2</math></p> <p><math>A = \pi r^2 = 9\pi</math></p>
--	--



26) A square and a semicircular region have the same perimeter. If the length of the radius of the semicircular region is 8, what is the length of one side of the square?

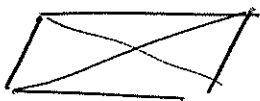
$$\begin{aligned} \text{Square} &= 8 + 8 + 8\pi \\ &= \frac{16 + 8\pi}{4} \\ &= 4 + 2\pi \end{aligned}$$

27) In the figure below, ABCD is a square. Points on each pair of adjacent sides of ABCD are connected to form 4 congruent right triangles with one leg two times as long as the other, as shown below. What fraction of the area of square ABCD is shaded?



### Properties of Quadrilaterals

1) Which of the following statements is **NOT** true about parallelograms?



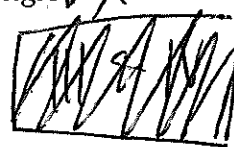
- a. consecutive angles are congruent ☒
- b. opposite sides are congruent ☒
- c. opposite angles are congruent ☒
- d. the diagonals bisect each other ☒

Both

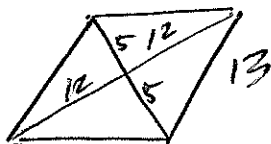
2) Which of the following quadrilateral is not a parallelogram?

- I. Trapezoid ☒
- II. Kite ☒
- III. Square ☒
- IV. Rectangle ☒

- A. I
- B. II and III
- C. I and II
- D. I, II, and III
- E. All of above



3) If one diagonal of a rhombus is 10 cm and the other is 24 cm.



a. How long is each side of the rhombus?

$$13 \text{ cm}$$

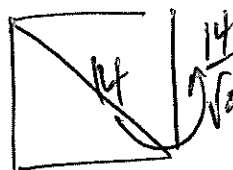
b. Find the perimeter.

$$13(4) = 52 \text{ cm}$$

c. Find the area.

$$10(24) \cdot \frac{1}{2} = 120 \text{ cm}^2$$

4) The diagonal of a square is 14 inches.



a. How long is each side of the square?

$$\frac{14}{\sqrt{2}} = 7\sqrt{2}$$

b. Find the perimeter.

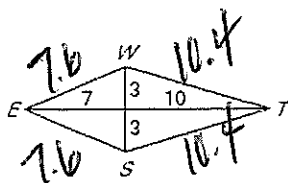
$$4(7\sqrt{2}) = 28\sqrt{2} \text{ in}$$

c. Find the area.

$$7(\sqrt{2}) \times 7(\sqrt{2}) = 49(2) = 98 \text{ in}^2$$

5) Use the figure below to:

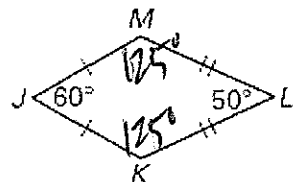
a. Find the length of each side.



b. Find the perimeter.  $74.1$

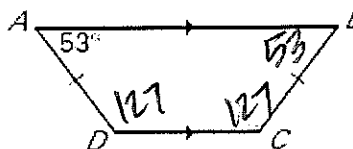
c. Find the area.  $\frac{1}{2}(6)(17) = 51$

6) Find the measurement for angles M and K.



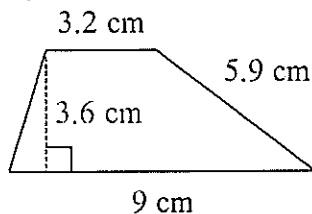
$$\frac{250}{2} = 125$$

7) Find the measurements of angle B, C, and D.



$$\frac{254}{2} = 127$$

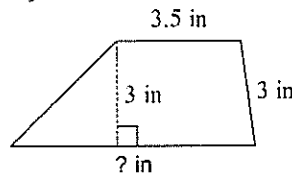
8) Find the area:



$$\frac{1}{2}(3.2+9) \cdot 3.6$$

$$21.96 \text{ cm}^2$$

9) Find the base:



Area = 15.6 in<sup>2</sup>

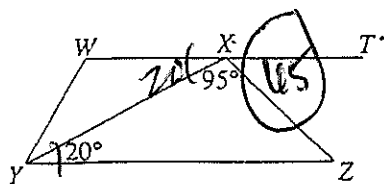
$$15.6 = \frac{1}{2}(3.5+x) \cdot 3$$

$$10.4 = 3.5 + x$$

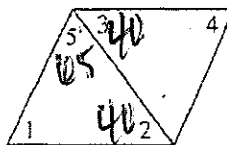
$$x = 6.9 \text{ in}$$

10)

In the figure below, WXZY is a trapezoid, point X lies on WT, and the angles are as marked. What is the measure of  $\angle ZXT$ ?

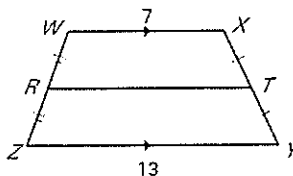


11) In the parallelogram below, a diagonal is shown and  $\angle 3$  measures  $40^\circ$  and  $\angle 5$  measures  $65^\circ$ . What is the  $m\angle 1$ ?



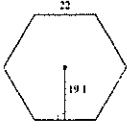
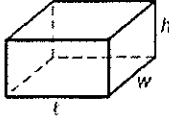
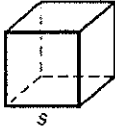
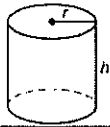
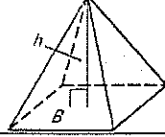
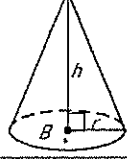

$$m\angle 1 = 75^\circ$$

12) Find the midsegment RT.

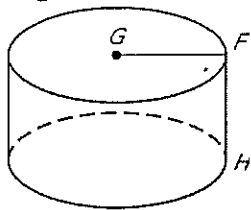


$$\frac{7+13}{2} = 10$$

→ find area of  $\Delta$ 's

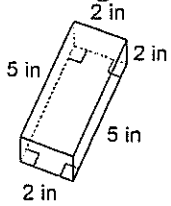
Solid	Formula: SURFACE AREA	Formula: VOLUME	Picture
REGULAR POLYGON	$2B + Ph$ area base perimeter height	$V = Bh$ area base	
RECTANGULAR PRISM	$2(lw) + 2(lh) + 2(wh)$	$V = lwh$	
CUBE	$6s^2$	$s^3$	
CYLINDER	$2\pi r^2 + 2\pi rh$	$\pi r^2 h$	
PYRAMID	$B + \frac{1}{2}Pl$ area base perimeter slant height	$\frac{1}{3}Bh$ area base	
CONE	$\pi r^2 + \pi rl$ area base circumference radius slant height	$\frac{1}{3}\pi r^2 h$	
SPHERE	$4\pi r^2$	$\frac{4\pi r^3}{3}$	

13) Use the diagram at the below to answer the questions at the right.



- Give the mathematical name of the solid. right cylinder
- What kind of figure is each base? circle
- Name the radius of the solid. GF
- Name the height of the solid. FH

14) Find the surface area and volume of the rectangular prism below.



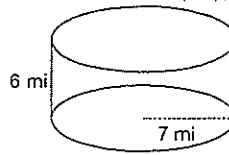
Surface Area:

$$2(2 \times 2) + 2(2 \times 5) + 2(2 \times 5)$$

Volume:

$$2 \times 2 \times 5 = 20 \text{ in}^3$$

15) Find the surface area and volume of the cylinder below and leave it in terms of  $\pi$ .



Surface Area:

$$2\pi r^2 + 2\pi rh$$

Volume:

$$\pi r^2 h$$

16) A cube has a volume of  $1728 \text{ cm}^3$ .

a. What is the area of one face of the cube?

b. What is the full surface area of the cube?

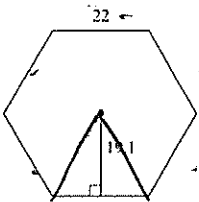
$$6(144) = 864 \text{ cm}^2$$

17) The surface area of one face of a cube is  $121 \text{ m}^2$ . Find the volume of the cube.

$$\sqrt{121} = 11 \quad (11)^3 = 1331 \text{ m}^3$$

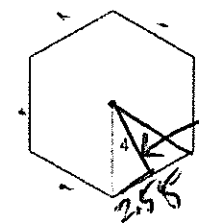
18) Find the area of the regular polygons below. All units are in inches.

a.



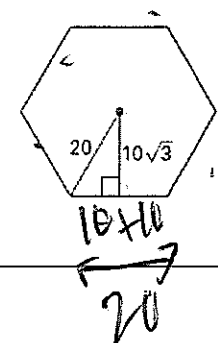
$$6 \left( \frac{1}{2} (22) (19.1) \right)$$

b. Find the area of the regular polygon below. Each side is 5 cm. Round to the nearest hundredth.



$$6 \times \left( \frac{1}{2} (5) (3.1) \right)$$

c.



$$6 \left( \frac{1}{2} (20) (10\sqrt{3}) \right)$$

19)

a. In #1 problem (a) I was given (circle 2):

Apothem "Radius" Side length

So I solved by finding area of one triangle & multiplying by 6

b. In #1 problem (b) I was given (circle 2):

Apothem "Radius" Side length

So I solved by Pythagorean theorem to find apothem and then

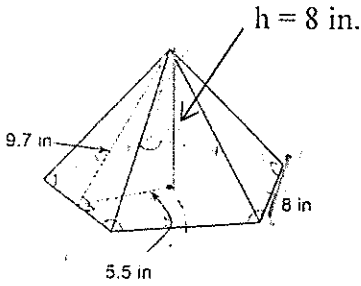
Continued →

a. In #1 problem (c) I was given (circle 2):

Apothem "Radius" Side length

So I solved by

20) Find the surface area and volume of the pyramid below.



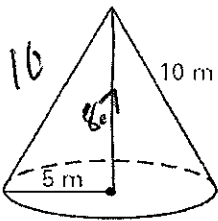
$$8 \cdot 9.7 \cdot \frac{1}{2} = 5(38.8) = 194 \text{ in}^2$$

$$\frac{1}{3} B h = \frac{1}{3} \cdot 5(5.5 \times 8) = \frac{1}{3} \cdot 220 \times 8 = \frac{1}{3} \cdot 1760 = 586.67 \text{ in}^3$$

21)

- Which of the values shown in # 3 is the **lateral** height of the pyramid?  
9.7
- Which of the following formulas requires **lateral** height for a pyramid—surface area or volume? surface area
- Which of the values shown in # 3 is the **actual** height of the pyramid?  
h = 8 in
- Which of the following formulas requires **actual** height for a pyramid—surface area or volume?  
volume

22) Find the surface area and volume of the cone below.



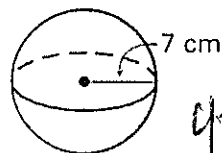
$$SA = \pi r^2 + \pi r l = \pi (5)^2 + \pi (5)(10) = 237.8 + 78.5 + 157.1$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (5)^2 (8.7) = 227.8 \text{ m}^3$$

23)

- Which of the values shown in # 5 is the **lateral** height of the cone?  
10 m
- Which of the following formulas requires **lateral** height for a cone—surface area or volume? surface area
- Which of the values shown in # 5 is the **actual** height of the cone?  
h = 8.7 (pythag thm)
- Which of the following formulas requires **actual** height for a cone—surface area or volume? volume

24) Find the surface area and the volume of the sphere below.



$$SA = 4\pi r^2 = 4\pi (7)^2 = 615.8 \text{ cm}^2$$

$$V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (7)^3 = 1436.8 \text{ cm}^3$$

25) State the error based on # 7.

$$V = \frac{4}{3} \pi (7)^2$$

$$V = \frac{4}{3} \pi (49)$$

$$V = 579.1 \text{ in}^3$$

should be cubed, not squared

Error:

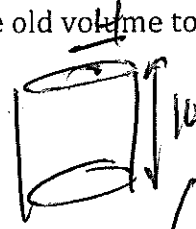
26) The length of each side of a cube is increased by 4 times. By how many times did the volume increase?

$$V = (4s)^3$$

$$= 64s^3$$

64 times

27) A cylinder has a radius of 4 ft and height of 10 ft. If the radius is increased by 2 times, find the ratio of the old volume to the new volume.



$$V = \pi r^2 h$$

$$= \pi (2r)^2 h$$

$$= 4\pi r^2 h$$

1:4

28) The length of each side of a cube is multiplied by 3. What is the change in volume of the cube?

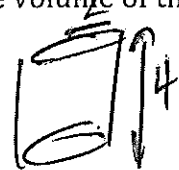
- A. The volume is 3 times greater.  
 B. The volume is 6 times greater.  
 C. The volume is 9 times greater.  
 D. The volume is 27 times greater.

$$V = s^3$$

$$V = (3s)^3$$

$$3^3 = 27$$

29) A cylinder has a height of 4 cm and a radius of 2 cm. If both the height and radius are doubled, what is the ratio of the volume of the original cylinder to the volume of the new cylinder?



$$V = \pi r^2 h$$

$$= \pi (2r)^2 \cdot 2h$$

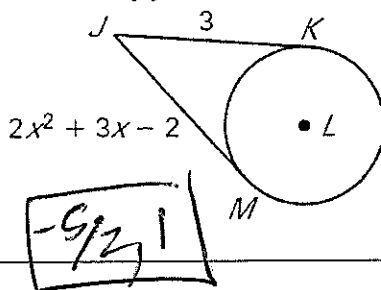
$$= \pi \cdot 4r^2 \cdot 2h$$

$$= 8\pi r^2 h$$

1:8

### Properties of Circles

1) The points K and M are points of tangency. Find the value(s) of x.



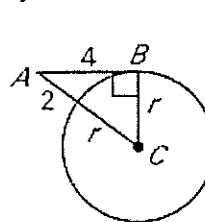
$$x^2 + 3x - 2 = 3$$

$$x^2 + 3x - 5 = 0$$

$$\frac{2x}{x} + \frac{5}{-1}$$

-5/2, 1

2) Find the radius, r.



$$4^2 + r^2 = (2+r)^2$$

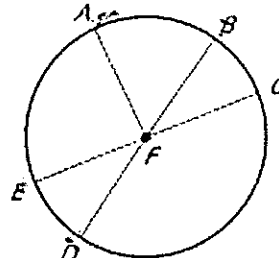
$$16 + r^2 = 4 + 4r + r^2$$

$$12 = 4r$$

r = 3

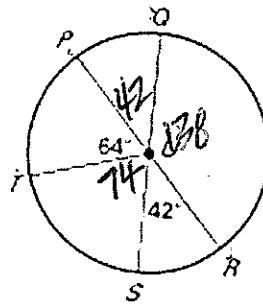
In  $\odot F$ , determine whether the given arc is a *minor arc*, *major arc*, or *semicircle*.

- $\widehat{AB}$  minor
- $\widehat{AE}$  minor
- $\widehat{EAC}$  semi
- $\widehat{ACD}$  major
- $\widehat{CAD}$  major
- $\widehat{DEB}$  semi
- $\widehat{BAE}$  minor
- $\widehat{DEC}$  major



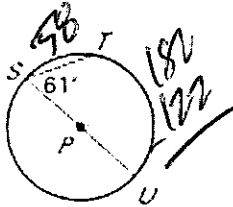
In the figure,  $PR$  and  $QS$  are diameters of  $\odot U$ . Find the measure of the indicated arc.

9.  $m\widehat{PQ}$  42  
 10.  $m\widehat{ST}$  74  
 11.  $m\widehat{PS}$  116  
 12.  $m\widehat{RT}$  116  
 13.  $m\widehat{QS}$  116  
 14.  $m\widehat{QR}$  180  
 15.  $m\widehat{PQS}$  116  
 16.  $m\widehat{TQR}$  116  
 17.  $m\widehat{PS}$  116  
 18.  $m\widehat{PTR}$  180

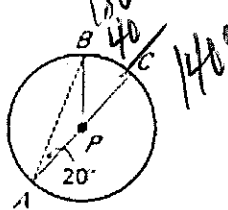


Find the measure of the indicated angle or arc in  $\odot P$ .

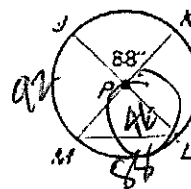
2.  $m\widehat{ST}$



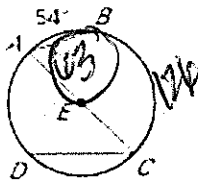
3.  $m\widehat{AB}$



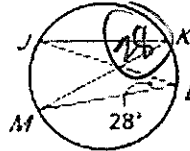
4.  $m\angle JLM$



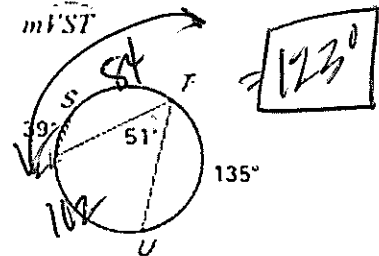
5.  $m\angle A$



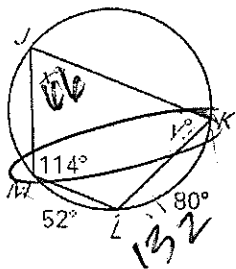
6.  $m\angle K$



7.  $m\widehat{ST}$

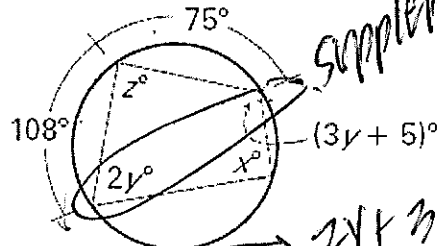


8) Find the values of the variables.



$$\begin{aligned} 180 \\ - 114 \\ \hline y = 66^\circ \\ x = 66^\circ \end{aligned}$$

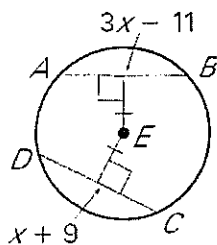
9) Find the values of the variables.



Supplementary

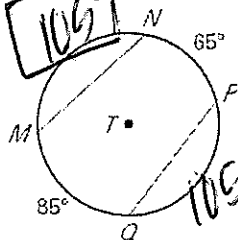
$$\begin{aligned} 2y + 3y + 5 &= 180 \\ 5y &= 175 \\ y &= 35^\circ \\ x &= 91.5^\circ \\ z &= 88.5^\circ \end{aligned}$$

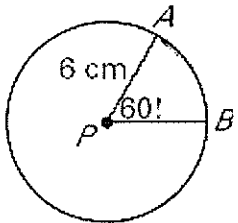
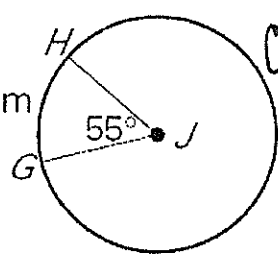
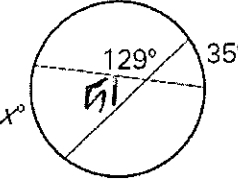
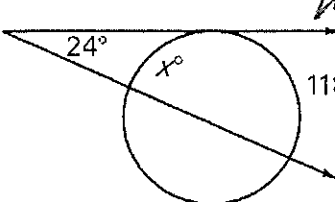
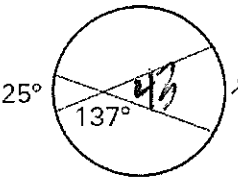
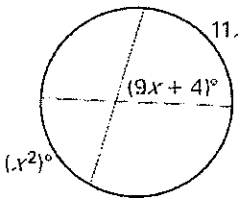
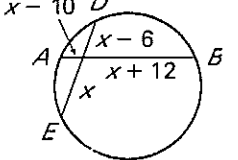
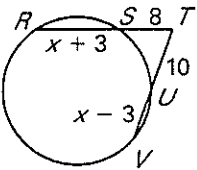
10) Find the value of x in the figure below.



$$\begin{aligned} 3x - 11 &= x + 9 \\ 2x &= 20 \\ x &= 10 \end{aligned}$$

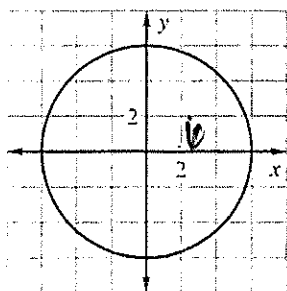
11) Find the measure of arc MN.



<p>12) Find the length of arc AB.</p>  <p>Handwritten work: <math>12\pi \left(\frac{60}{360}\right)</math>  <math>\boxed{6.28 \text{ cm}}</math></p>	<p>13) Find the circumference of circle J.</p>  <p>Handwritten work: <math>C\left(\frac{55}{360}\right) = 19.71</math>  <math>\boxed{C = 129.01 \text{ cm}}</math></p>
<p>14) Find the value of x.</p>  <p>Handwritten work: <math>51 = \frac{1}{2}(35 + x)</math>  <math>102 = 35 + x</math>  <math>\boxed{x = 67}</math></p>	<p>15) Find the value of x.</p>  <p>Handwritten work: <math>24 = \frac{1}{2}(118 - x)</math>  <math>48 = 118 - x</math>  <math>\boxed{x = 70}</math></p>
<p>16) Find the value of x.</p>  <p>Handwritten work: <math>43 = \frac{1}{2}(25 + x)</math>  <math>86 = 25 + x</math>  <math>\boxed{x = 61}</math></p>	<p>17) Find the value of x.</p>  <p>Handwritten work: <math>9x + 4 = \frac{1}{2}(11x + x^2)</math>  <math>18x + 8 = 11x + x^2</math>  <math>= x^2 - 7x + 8</math>  <math>(x+1)(x-8)</math>  <math>\boxed{x = 8}</math></p>
<p>18) Find x:</p>  <p>Handwritten work: <math>x(x-10) = (x-10)(x+12)</math>  <math>x^2 - 10x = x^2 + 12x - 10x - 120</math>  <math>18x</math>  <math>120 = 8x</math>  <math>\boxed{x = 15}</math></p>	<p>19) Find x:</p>  <p>Handwritten work: <math>8(11+x) = 10(x+7)</math>  <math>88 + 8x = 10x + 70</math>  <math>18 = 2x</math>  <math>\boxed{x = 9}</math></p>

Write the standard equation of the circle.

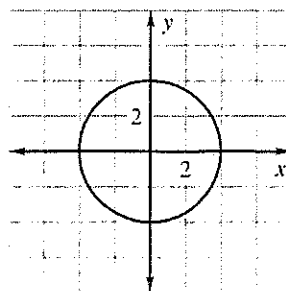
1.



$$x^2 + y^2 = 6^2$$

$$x^2 + y^2 = 36$$

2.



$$x^2 + y^2 = 4^2$$

$$x^2 + y^2 = 16$$

Write the standard equation of the circle with the given center and radius.

5. Center (0, 0), radius 9

$$x^2 + y^2 = 9^2$$

$$x^2 + y^2 = 81$$

6. Center (1, 3), radius 4

$$(x-1)^2 + (y-3)^2 = 16$$